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Modeling of Wave Propagation in Dispersive Media Using New ADE-TLM Method

El Hadi El Ouardy*, Hamid Bezzout, and Hanan El Faylali

Department of physics
Laboratory of Electronic Systems Information Processing Mechanics and Energetics
Ibn Tofail University, B.P: 242 Kenitra, Morocco

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INTRODUCTION

❖ We introduce a novel approach for modeling dispersive media by employing the Transmission Line Matrix (TLM) technique enhanced with the symmetrical condensed node (SCN-TLM) technique. This method effectively simulates electromagnetic wave interactions with Lorentz and Drude media through a scattered-field formulation.

❖ The Transmission Line Matrix (TLM) approach was designed to present a direct comparison between the fundamentals of electromagnetic wave

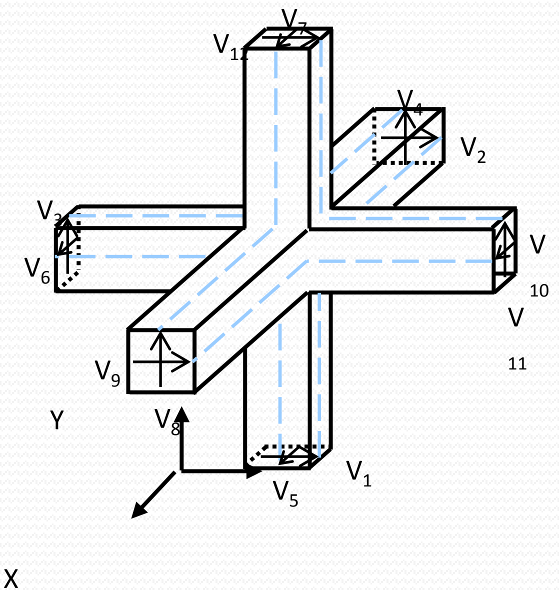
❖ In recent years, we report a many TLM-based approaches for the analysis of a dispersive medium.

RELATED WORK

❖ The auxiliary differential equation (ADE-TLM) technique is employed to model chiral media, Lorentz and Drude media, by applying the ADE approximation, and it is additionally used to represent Cole-Cole media.

❖ This paper introduces a novel advancement in the ADE-TLM framework, offering an innovative formulation designed for modeling environments characterized by Lorentz and Drude properties.

❖ The SCN-TLM, it employs 12 principal ports to model free space and incorporates 3 additional ports as voltage sources to characterize the properties of Drude and Lorentz media



METHODOLOGY

- ❖ We delve into the innovative approach of modeling Lorentz and Drude media through the New Auxiliary Differential Equation-Transmission Line Matrix (ADE-TLM) algorithm.

- ❖ This novel technique, aimed at accurately representing dispersive media interactions, involves a sophisticated formulation incorporating both the Lorentz and Drude media characteristics.

- ❖ We have devised a comprehensive algorithm capable of capturing the intricate behaviors of both Lorentz and Drude media, offering a robust foundation for our subsequent analyses and simulations.

FORMULATIONS AND EQUATIONS

➢The Auxiliary Differential Equation (ADE-TLM) :

technique is employed to model Lorentz and Drude media by applying the ADE approximation.

➢ The magnetic and electric fields are separated into incident and scattered terms :

$$E_{total} = E_{inc} + E_{scat} \qquad H_{total} = H_{inc} + H_{scat}$$

➢The Maxwell equations for the polarization current density :

$$\nabla \times E = -\mu_0 \frac{\partial H}{\partial t}$$

$$\nabla \times H = \varepsilon_0 \varepsilon_\infty \frac{\partial E}{\partial t} + \sum_{p=1}^{p} J_p$$

LINEAR DRUDE

The polarization current density:

$$\frac{\partial^2 J_p}{\partial t^2} + \tau_p \frac{\partial J_p}{\partial t} = \varepsilon_0 \omega_p^2 \frac{\partial E}{\partial t}$$

The updated equation :

$$J_p^{n+1} = a_d J_p^n + b_d \left(E^{n+1} + E^n \right)$$

$$a_d = \frac{2 - \tau_p \Delta t}{2 + \tau_p \Delta t}$$

$$b_d = \frac{\varepsilon_0 \omega_p^2 \Delta t}{2 + \tau_p \Delta t}$$

the polarization current : $$J_p^{n+\frac{1}{2}} = \frac{1}{2}\left(J^{n+1} + J^{n}\right)$$

We obtain : $$J_p^{n+\frac{1}{2}} = \frac{1}{2}\left[\left(1+a_d\right)J_p^n + b_d\left(E^{n+1}+E^n\right)\right]$$

The temporal discretization at step n+1/2:

$$\nabla \wedge H^{n+\frac{1}{2}} = \varepsilon_0\varepsilon_\infty\left(\frac{E^{n+1}-E^n}{\Delta t}\right) + \frac{1}{2}\left[\left(1+a_d\right)J_p^n + b_d\left(E^{n+1}+E^n\right)\right]$$

We obtain :

$$E^{n+1} = \left(\frac{2\varepsilon_0\varepsilon_\infty - \Delta t b_d}{2\varepsilon_0\varepsilon_\infty + \Delta t b_d}\right)E^n + \left(\frac{2\Delta t}{2\varepsilon_0\varepsilon_\infty + \Delta t b_d}\right)\left(\nabla \wedge H^{n+\frac{1}{2}} - \frac{1}{2}\left(1+a_d\right)J_p{}^n\right)$$

We convert the electric field to a voltage : $E^n = \frac{V^n}{\Delta l}$

we obtain : $$V^{n+1} = \left(\frac{2\varepsilon_0\varepsilon_\infty - \Delta t b_d}{2\varepsilon_0\varepsilon_\infty + \Delta t b_d}\right)V^n + \left(\frac{2\Delta t \Delta l}{2\varepsilon_0\varepsilon_\infty + \Delta t b_d}\right)\left(\nabla \wedge H^{n+\frac{1}{2}} - \frac{1}{2}(1+a_d)J_p^{\ n}\right)$$

$$\nabla \wedge H^{n+\frac{1}{2}} = \frac{\varepsilon_0}{2\Delta t \Delta l}\left[\sum V_i^{\ n+1} - \sum V_i^{\ n} - V_{sx}^{\ n}\right]$$

The total voltage : $$\begin{pmatrix} V_x^{n+1} \\ V_y^{n+1} \\ V_z^{n+1} \end{pmatrix} = \begin{pmatrix} \frac{2}{4+Y_{ox}} \\ \frac{2}{4+Y_{oy}} \\ \frac{2}{4+Y_{oz}} \end{pmatrix}\begin{pmatrix} \left[V_1^i + V_2^i + V_9^i + V_{12}^i + \frac{1}{2}V_{sx}\right]^{n+1} \\ \left[V_3^i + V_4^i + V_8^i + V_{11}^i + \frac{1}{2}V_{sy}\right]^{n+1} \\ \left[V_5^i + V_6^i + V_7^i + V_{10}^i + \frac{1}{2}V_{sz}\right]^{n+1} \end{pmatrix}$$

The normalized admittances : $$Y_{ou} = 4\left(\frac{2\varepsilon_0\varepsilon_\infty + \Delta t b_d}{2\varepsilon_0} - 1\right)$$

the voltage sources : $$V_{su}^{n+1} = -V_{su}^{\ n} - 4\left[\frac{\Delta t b_d}{\varepsilon_0}V_u^n + \frac{\Delta t \Delta l}{2\varepsilon_0}(1+a_d)J_p^{\ n}\right]$$

LINEAR LORENTZ

The polarization current density:

$$\omega_p^2 J_p + 2\delta_p \frac{\partial J_p}{\partial t} + \frac{\partial^2 J_p}{\partial t^2} = \varepsilon_0 \Delta\varepsilon_p \omega_0^2 \frac{\partial E}{\partial t}$$

We obtain by using the finite difference time :

$$\omega_p^2 \left(\frac{J_p^{n+1} + J_p^n}{2} \right) + 2\delta_p \left(\frac{J_p^{n+1} - J_p^{n-1}}{2\Delta t} \right) + \left(\frac{J_p^{n+1} - 2J_p^n + J_p^{n-1}}{(\Delta t)^2} \right) = \varepsilon_0 \Delta\varepsilon_p \omega_p^2 \left(\frac{E^{n+1} - E^{n-1}}{2\Delta t} \right)$$

The updated equation :

$$J_p^{n+1} = \alpha_p J_p^n + \xi_p J_p^{n-1} + \gamma_P \left(\frac{E^{n+1} - E^{n-1}}{2\Delta t} \right)$$

$$\alpha_p = \frac{2 - \omega_L^2 (\Delta t)^2}{1 + \delta_L \Delta t} \qquad \xi_p = \frac{\delta_L \Delta t - 1}{1 + \delta_L \Delta t} \qquad \gamma_P = \frac{\varepsilon_0 \Delta\varepsilon_p \omega_L^2 (\Delta t)^2}{1 + \delta_L \Delta t}$$

The time discretization:

$$E^{n+1} = E^{n} + \frac{\Delta t}{\varepsilon_0}\left(\nabla \wedge H^{n+\frac{1}{2}} - J^{n+\frac{1}{2}}\right)$$

We convert the electric field to a voltage:

$$E^{n} = \frac{V^{n}}{\Delta l}$$

we find:

$$J_p^{n+1} = \alpha_p J_p^{n} + \xi_p J_p^{n-1} + \gamma_P\left(\frac{V^{n+1} - V^{n-1}}{2\Delta l \Delta t}\right)$$

$$V^{n+1} = V^{n} + \frac{\Delta l \Delta t}{\varepsilon_0}\left(\nabla \wedge H^{n+\frac{1}{2}} - J^{n+\frac{1}{2}}\right)$$

We obtain :

$$J_p^{n+1} = \alpha_p J_p^{n} + \xi_p J_p^{n-1} + \gamma_P\left(\frac{V^{n} - V^{n-1}}{\Delta l \Delta t}\right)$$

The SCN-TLM method:

$$\begin{pmatrix} \nabla \wedge H_x^{n+\frac{1}{2}} \\ \nabla \wedge H_y^{n+\frac{1}{2}} \\ \nabla \wedge H_z^{n+\frac{1}{2}} \end{pmatrix} = \begin{pmatrix} \frac{\varepsilon_0}{2\Delta t \Delta l}\left[\left(V_1^i + V_2^i + V_9^i + V_{12}^i\right)^{n+1} - \left(V_1^r + V_2^r + V_9^r + V_{12}^r\right)^{n}\right] \\ \frac{\varepsilon_0}{2\Delta t \Delta l}\left[\left(V_3^i + V_4^i + V_8^i + V_{11}^i\right)^{n+1} - \left(V_3^r + V_4^r + V_8^r + V_{11}^r\right)^{n}\right] \\ \frac{\varepsilon_0}{2\Delta t \Delta l}\left[\left(V_5^i + V_6^i + V_7^i + V_{10}^i\right)^{n+1} - \left(V_5^r + V_6^r + V_7^r + V_{10}^r\right)^{n}\right] \end{pmatrix}$$

Applying the charge conservation's laws:

$$\begin{pmatrix} V_1^r + V_2^r + V_9^r + V_{12}^r \\ V_3^r + V_4^r + V_8^r + V_{11}^r \\ V_5^r + V_6^r + V_7^r + V_{10}^r \end{pmatrix}^n = \begin{pmatrix} V_1^i + V_2^i + V_9^i + V_{12}^i \\ V_3^i + V_4^i + V_8^i + V_{11}^i \\ V_5^i + V_6^i + V_7^i + V_{10}^i \end{pmatrix}^n + \begin{pmatrix} V_{sx} \\ V_{sy} \\ V_{sz} \end{pmatrix}^n$$

the total voltage:

$$\begin{pmatrix} V_x^{n+1} \\ V_y^{n+1} \\ V_z^{n+1} \end{pmatrix} = \begin{pmatrix} \frac{2}{4+Y_{ox}} \\ \frac{2}{4+Y_{oy}} \\ \frac{2}{4+Y_{oz}} \end{pmatrix} \begin{pmatrix} \left[V_1^i + V_2^i + V_9^i + V_{12}^i + \frac{1}{2} V_{sx} \right]^{n+1} \\ \left[V_3^i + V_4^i + V_8^i + V_{11}^i + \frac{1}{2} V_{sy} \right]^{n+1} \\ \left[V_5^i + V_6^i + V_7^i + V_{10}^i + \frac{1}{2} V_{sz} \right]^{n+1} \end{pmatrix}$$

The normalized admittances : $Y_{ou} = 4(\varepsilon_\infty - 1)$

the voltage sources : $V_{su}^{n+1} = -V^n{}_{su} - \frac{\Delta l \Delta t}{2\varepsilon_0} \sum_{p=1}^{p} \left(J_{pu}^{n+1} + J_{pu}^{n} \right)$

RESULTS

To evaluate the efficiency and validity of the New ADE-TLM algorithm which includes voltage sources for the Lorentz and Drude media.

The TLM network is divided in $(1,1,1000)\Delta l$

the time step : $\Delta t = 0.4166$ ps

the space step : $\Delta l = 250\mu m$

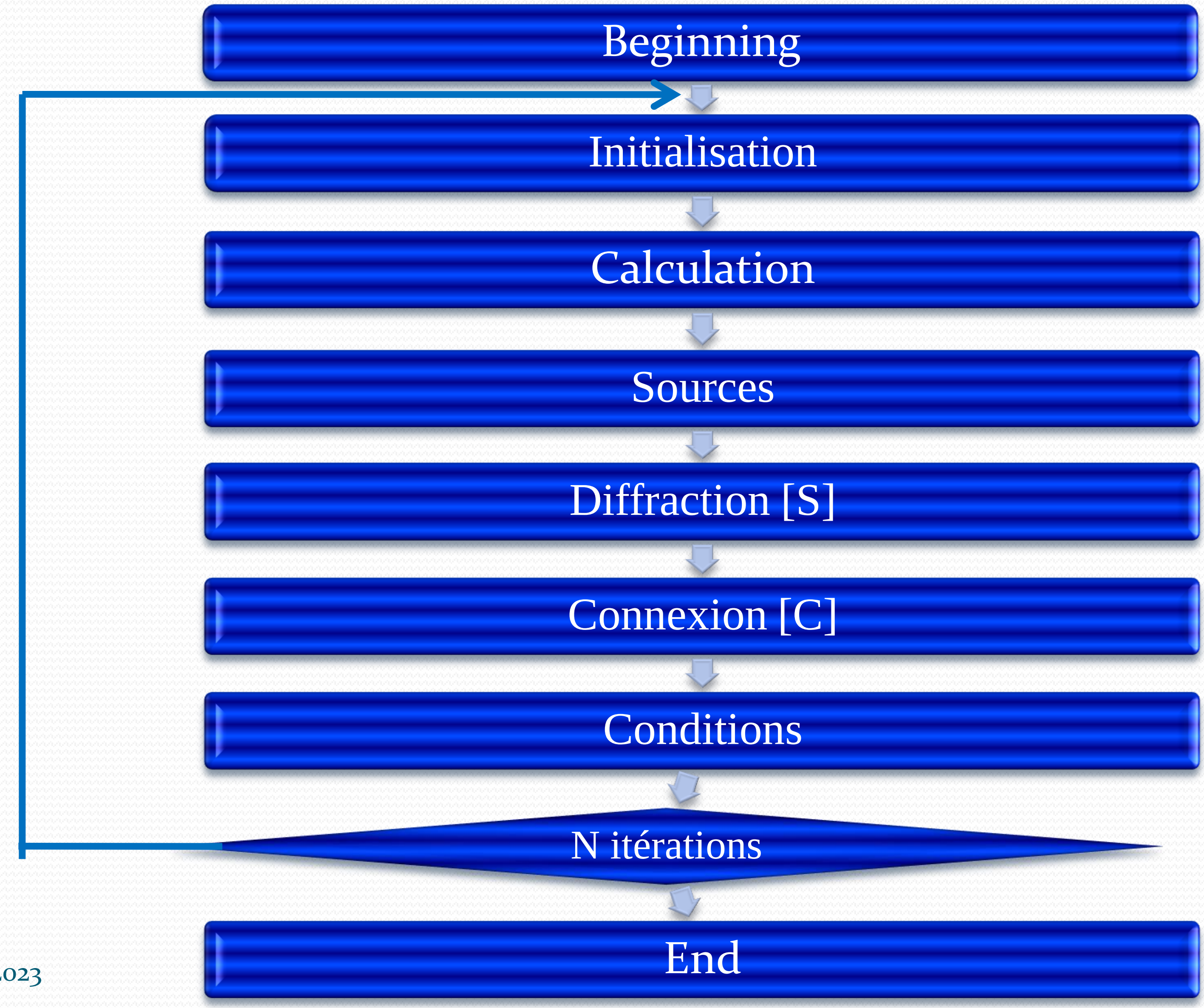
the following parameters describe the Lorentz media:

$\varepsilon_s = 3.0$ $\varepsilon_\infty = 1.5$ $\omega_0 = 2\pi \times 25GHz$ $\delta_L = 0.1\omega_0$

the Drude half space :

$\varepsilon_s = 3,0$ $\varepsilon_\infty = 1.0$ $\tau_0 = 2 \times 10^{10} s$

Flowchart of the TLM method



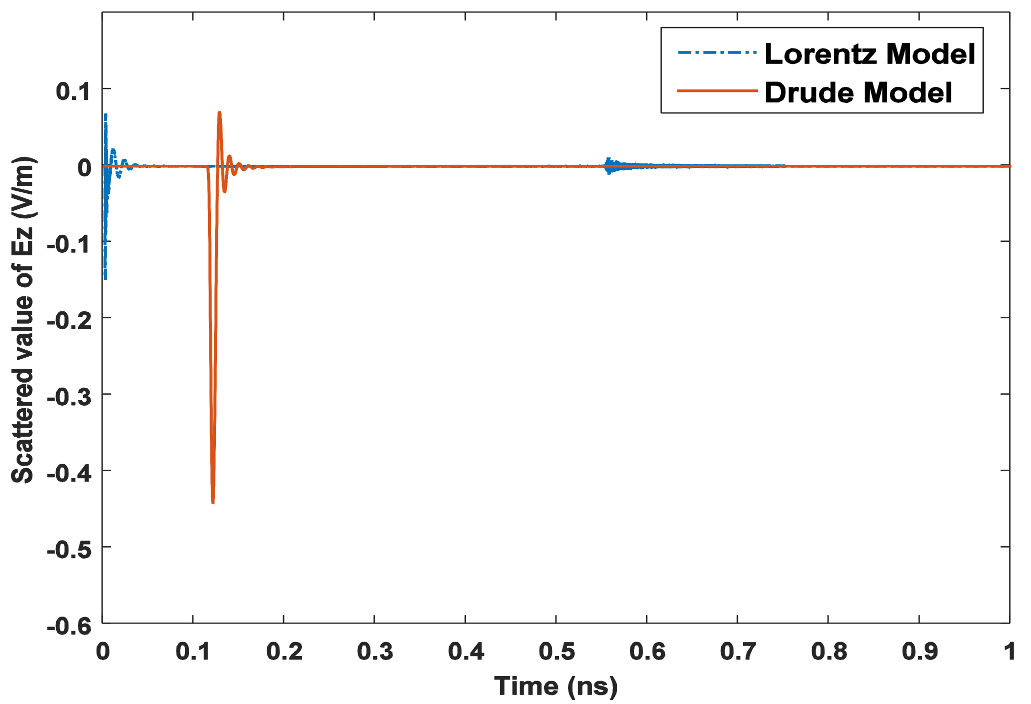


Figure 1 shows waveforms calculated using the New ADE-TLM method. These waveforms show the behavior in the free-space of the scattered electric fields when interacting with both the Lorentz and Drude half-space models.

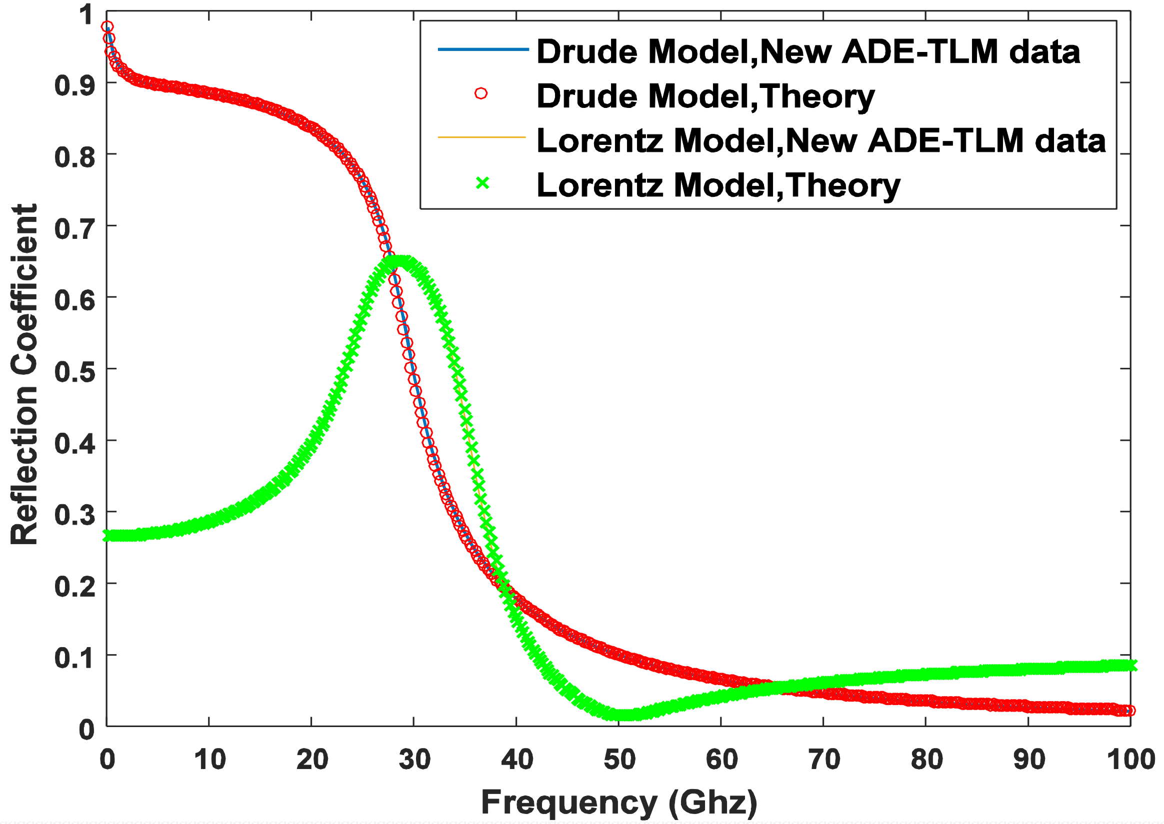


Figure 2 Compares the reflection coefficient for both the Lorentz and Drude instances utilizing New ADE-TLM and accurate analytical results.

DISCUSSION

An efficient agreement was noted between the exact theoretical results as defined by Eq. (24) and the new ADE-TLM results for both the Lorentz and Drude examples. This result agrees very well [21].

$$\left|\Gamma(\omega)\right| = \left|\frac{\sqrt{\varepsilon_0} - \sqrt{\varepsilon * (\omega)}}{\sqrt{\varepsilon_0} + \sqrt{\varepsilon * (\omega)}}\right|$$

CONCLUSION

❖ In this study, we provide a novel ADE-TLM approach for modeling Lorentz and Drude Dispersive media. This approach employs the connection between centered derivatives approximations, polarization current density J, and electrical voltage.

❖ The results obtained utilizing our New ADE-TLM method are in excellent agreement with the analytical values of the reflection coefficient demonstrating the validity of the proposed method.

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